Synopsis

‘STUDY OF INTEGRAL TRANSFORM ON TIME SPACE AND IT’S APPLICATIONS’

Submitted to

Swami Ramanand Teerth Marathwada University,
Nanded- 431606.

For the Degree of

Doctor of Philosophy (Ph.D.)

In Mathematics

(Faculty of Science and Technology)

Submitted by

Mr. Santosh Keshavrao Bokare

(Research Scholar)

Under the Guidance of

Dr.S.B.Chavhan

(Research Supervisor)

Head, Dept. of Mathematics

Digambarrao Bindu ACS College, Bhokar,

Dist. Nanded, (Maharashtra).

December 2020

To,

The Deputy Registrar (P.G. Section),

S. R. T. M. U.,

Nanded.

Subject: **Permission to submit the Synopsis for Doctor of Philosophy (**Ph. D.**) in Mathematics.**

Respected Sir,

I am recognized guide in Mathematics of our University S. R. T. M. U., Nanded. My candidate **Mr. Santosh Keshavrao Bokare** is submitting registration form for Ph. D. under my guidance. So kindly permit to submit the Synopsis.

All required and needful documents will be fulfilled by my candidate.

Thanking you.

Yours Faithfully

Dr.S.B.Chavhan

Head, Dept. of Mathematics
Digambarrao Bindu ACS College,Bhokar,
Dist. Nanded, (Maharashtra).

To,
The Director,

School of Mathematical Sciences,

S.R.T.M.University,

Nanded.

Subject: Permission to Submit Registration Form of Mr. Santosh Keshavrao Bokare.

Respected Sir,

With reference to subject cited above, sir, I am recognized guide in Mathematics of our research center. My candidate Mr. Santosh Keshavrao Bokare is submitting registration form for Ph. D. under my guidance through our research center.

So kindly permit to submit the form.

All required needful documents will be fulfill by my candidate.

Thanking you.

Yours Faithfully

Dr.S.B.Chavhan

Head, Dept. of Mathematics
Digambarrao Bindu ACS College,Bhokar,
Dist. Nanded, (Maharashtra).

Dist. Nanded, (Maharashtra).

‘STUDY OF INTEGRAL TRANSFORM ON TIME SPACE AND IT’S APPLICATION’

1) Introduction:

The Study of integral transforms in various countries in the world. Namians 1980 has 1st time introduced fractional Fourier transform in the generalized classical transform by using eigen value method. The Natural language for generalized function and singular function like Dirac-Delta was first described by Zemanian [12],[13]. Fractional Fourier transforms is particularly use in optics and images processing. Pei and Ding[9],[10] have remarkable contribution in this field and have done work in majority of integral transforms along with their important applications. The Fourier transform is undoubtly one of the most valuable and frequently used tools in signal processing and analysis. Fourier transform is not only use technical subject but also in other subject like music share market psychology, economics etc. but all scientist are take small domain but our problem we take enlarge domain to obtain various result for our integral transform and try to obtain many application in given transform.

No doubt Fourier transform has a special place among these integral transforms due to its innumerous field of applications mainly in optics, signal processing, image processing etc. A much more general integral transform namely linear canonical transform was introduced in 1970 by Moshinky and Quesne [7]. Namias [8] introduced the fractional Fourier transform, using method of eigen values and eigen functions. Pie and Ding have defined fractional cosine transform and fractional sine transform, from the definition of fractional Fourier transform and also introduced canonical cosine transform and canonical sine transform as generalization of these fractional cosine transform and fractional sine transform. The significance of the integral transforms is that they provide useful operational methods for solving initial value problems and initial boundary value problems for the linear differential and integral equation. The main influences for the development of operational calculus of the

integral transform was the study of differential and integral equations arising in the applied mathematics, mathematical physics and engineering sciences. In this investigation we will study New transforms by using new techniques and their properties, applications and pictotorial representation using Matlab software.

Review of Literature :-

In the literature there are several integral transforms that are widely used in physics, astronomy as well as in engineering . Watunga [1,2] introduce a new integral transform and named sumudu transform that is defined by the formula

$$\boldsymbol{f(u) = S[f(t); u] = \frac{1}{u}\int_0^{\infty} e^{-\frac{t}{u}} f(t)dt \ , u \in (\tau_1, \tau_2),} \qquad (\mathbf{1.1})$$

And applied it to find the solution of ordinary differential equation in control engineering problems. It appeared like the modification of well known Laplace transform L[f(t);u], where

$$\boldsymbol{L[f(t); u] = \int_0^{\infty} e^{-ut} f(t)dt.} \qquad (\mathbf{1.2})$$

However in [3,4], some fundamental properties of sumudu transform were established by looking at the properties of this transform one can notice that the sumudu transform has very special and useful properties and it can help with intricate applications in the sciences and engineering. For example, in[5],in sumudu transform was extended to distributions(generalized functions) and some of their properties were also studied in [6,7].Recently Kilicman applied this transform to solve a system of differential equations, see[8].Further in[9],a system of fractional linear differential equations where solved analytically by using a new method which was named fractional sumudu transform.We note that by using the sumudu transform technique we can reduce the computational work as compare to classical methods while still maintaing the high accuracy of numerical result. we also note that an interesting fact about the sumudu transform is that the original function and its sumudu transform have the same coefficients except for the factor n!, thus if $\boldsymbol{f(t) = \sum_{n=0}^{\infty} a_n t^n}$,then $\boldsymbol{F(u) = \sum_{n=0}^{\infty} n!\, a_n t^n}$ see [10]. Furthermore, Laplace and sumudu transforms of the Diracdelta function and the Heaviside function satisfy,

$$\boldsymbol{S_2[H(x,t) = \varepsilon_2[\delta(x,t)] = 1}\ ,$$

And

$$\boldsymbol{S_2[\delta(x,t)] = \varepsilon_2[H(x,t)] = \frac{1}{uv}}$$

In this study, authors purpose is to introduce the sumudu transform on a time scale and show the applicability of this interesting new transform and its efficiency in solving the linear system of dynamic equations and integral equations. Assume that T is a time scale such that sup T= ∞ and fix $\boldsymbol{t_0} \in \boldsymbol{T}$.

Let $\boldsymbol{z} \in \boldsymbol{R}$ ($\boldsymbol{the\ set\ of\ regressive\ functions}$), then $\boldsymbol{\Theta z} \in \boldsymbol{R\ and\ e_{\Theta z}(t,t_0)}$ is well defined. The Laplace transforms of the function $\boldsymbol{f}: \boldsymbol{T} \longrightarrow \boldsymbol{R}$ was defined by,

$$\boldsymbol{L\{f\}(z) = \int_{t_0}^{\infty} f(t)e_{\Theta z}(t,t_0) \triangle t} \qquad (\mathbf{1.3})$$

For $\boldsymbol{z} \in \boldsymbol{\Omega}\{\boldsymbol{f}\}, \boldsymbol{where\ \Omega}\{\boldsymbol{f}\}$ consist of all complex numbers $\boldsymbol{z \epsilon R}$ for which the improper integral exists.

The origin of the Fourier integral transformation can be traced to Fourier's celebrated work on the "Analytical theory of heat", which appeared in 1822. Fourier's major findings was to show that an arbitrary function defined on a finite interval could be expanded in trigonometric series which now bears his name. The Fourier integral transform of a function f(t) is defined by,

$$F(W) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t)e^{iwt}dt$$

wherever the integral exists.

Fourier transform was found to be important tool in signal processing, optics and almost all branches of science.

In the Fourier transform the original function is multiplied by, $e^{ix\xi} = cosx\xi + isinx\xi$ and the product is integrated. In the cosine, sine and Hartly transform, the exponential factor is replaced by,

$cosx\xi = Re\left(e^{ix\xi}\right),\ \ sinx\xi = Im\left(e^{ix\xi}\right), casx\xi = cosx\xi + sinx\xi$ respectively.
These classical transforms are therefore used when the signal is real.

Thus Fourier cosine $F_C(W)$ and Fourier sine $F_S(w)$ transforms of f(t) are defined as,

$$F_C(W) = \sqrt{\frac{2}{\Pi}}\int_0^\infty f(t)(coswt)dt, F_S(w) = \sqrt{\frac{2}{\pi}}\int_0^\infty f(t)sinwt\ dt.$$

And the Hartley transform are f(t) is defined by,

$$H(W) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(t)cas(wt)dt$$

Fourier cosine transform, Fourier sine transform, Hartley transform and Mellin transform are said to be integral transform in Fourier class.

In the day to day examples, number of situations are modeled mathematically as differential equations and are solved by the integral transform method. The method is also useful while solving boundary value problem because this method transforms the problem from one setting to another where it can be easily solved.

The impact of the generalized function on the integral transformation has recently enriched the theory of both branches. Zemanian, Zayed [11] had made significant contributions to the theory of generalized integral transformations.

The idea of fractional power of Fourier operator appeared in mathematical literature in 1929 . But seeing the significant impact of fractional Fourier transform in all areas of sciences and engineering e.g. in signal processing, Optics and Quantum mechanics [1],[2],[3]. It is flourished in the last two decade.

In this work we decided to study generalization of Convectional integral transform such as S-Transform , n-dimensional canonical cosine transforms, n-dimensional canonical sine transforms, Gabor transform and wavelet transform, to the generalized sense. We shall extend to generalized functions of Zemanian type by kernel method for which proper testing functions space will be defined analyticity of the transforms will discussed any

integral transform is only considered as complete, if its inverse is given. So “inversion theorem” will be proved. Some operation transform formulae will be obtained for both transforms. And also we propose to do a systematic study of various aspects of canonical transforms such n-dimensional canonical cosine transforms, n-dimensional canonical sine transforms. Lastly applications will be given along with their values for some special functions. We shall discuss application of integral transforms of generalized function in several fields.

2) <u>Methodology</u> :-

- In the present investigations, books and journals will be used to collect the historical data for this descriptive study. Simultaneously recent information obtained from Peis, Ding & others for development of this problem.
- Internet surfing will be very useful for getting some references and knowing latest position of this problems in research field.
- Use of computer and high-end software like Matlab/SCILAB will also be done to obtain and /or verify as many results as possible.

3) Objectives:

- The research work turned out has the potential of distribution theory, the theory of integral transforms, studying their topological and distributional properties to solve engineering problems. The confluence of these theories leads to more significant and useful applications such as real life, biological sciences, signal processing, radar system etc.
- To do the study of some integral transforms on time scales, boundary value problems and Functional analysis.
- The aim to obtain inversion theorem, basic properties like modulation linearity and applications to partial differential equation of New transforms by utility of enlarge domain.
- In general, integral transforms and graphical transforms have some limitations. In this Problem, utility will be enlarged to domain of transforms to solve most differential equations and integral equations will be easily explained by present problem on time scale .
- To solve serval dynamical equation for a given integral transform on Time scale.
- To introduce some basic integral transforms on time scale T.
- Attempts to achieve some application in real life, engineering problems and biological sciences will be undertaken.

We hope that the work will definitely add to the area of generalized functions and integral transforms both.

4) International status:

The Sumudu transform was extended to distributions (generalized functions) and some of their properties were also studied Kilicman,A,Eltayeb. Recently Kilicman applied this transform to solve the system of a differential equation see[16]. Further a system of fractional linear differential equation were solved analytically by using a new method which was named fractional Sumudu transform .

Wavelet analysis is originally introduced in order to improve seismic signal analysis by switching from shortime Fourier analysis to new better algorithms to detect and analyze abrupt changes in signals Daubechies, Mallat. In time-frequency analysis of a signal, the classical Fourier transform analysis is inadequate because Fourier transform of a signal does not contain any local information. This is the major drawback of the Fourier transform. To overcome this drawback, Dennis Gabor in 1946, first introduced the windowed-Fourier transform, i.e. short- time Fourier transform known later as Gabor transform. Meyer found the existing literature of wavelets. Later many eminent mathematicians e.g. I. Daubechies, A. Grossmann, S. Mallat, Y. Meyer, R. A. deVore, Coifman, V. Wickerhauser, Lohaman A.[6] made a remarkable contribution to the wavelet theory. The modern applications of wavelet theory as diverse as wave propagation, data compression, signal processing, image processing, pattern recognition, computer graphics, the detection of aircraft and submarines, improvement of CAT scans and some other medical image technology etc. In this study, our main goal is to find out the advantages of wavelet transform compared to Fourier transform. The Study of integral transforms in various countries in the world. Namians 1980 has 1st time introduced fractional Fourier transform in generalized the classical transform by using eigen value method. The Natural language for generalized function and singular function like Dirac-Delta was first described by Zemanian. Fractional Fourier transforms is particularly use in optics and images processing. Pei and Ding have remarkable contribution in this field and have done work in majority of integral transforms along with their important applications. The Fourier transform is undoubtly one of the most valuable and frequently used tools in signal processing and analysis. Fourier transform is not only use technical

subject but also in other subject like music share market psychology, economics etc. but all scientist are take small domain but our problem we take enlarge domain to obtain various result for our integral transform and try to obtain many application in given transform.

5) National status:

Integral transform play an important role in the mathematics, Physics and engineering mathematics many application of integral transform is useful in national development like as radar system, optical communication, quantum mechanics, signal processing etc. Some limitation of the present situation that's why we describe a new integral transform by using enlarge domain so we obtain various method to for solving differential equation and obtain large number of application in that transforms. C. Saritha, V. Sukanya, Y. Narasimha Murthy, B.R.Bhonsale[4], K.M.Saksena, S.B.Chavhan [5], R.S. Pathak, Shri Karunakaran, P.K.Banerji etc considered various integral transforms and their generalizations to generalized functions and used them for solving boundary value problems. This paper deals with the study of ECG signals using wavelet transform analysis. In the first step an attempt was made to generate ECG waveforms by developing a suitable MATLAB simulator and in the second step, using wavelet transform, the ECG signal was de noised by removing the corresponding wavelet coefficients at higher scales. Then QRS complexes were detected and each complex was used to find the peaks of the individual waves like P and T, and also their deviations.

6) Significant outcome from Proposed Work :

The role and significance of the present study is in Mathematics, Physics and Engineering Mathematics. Many applications of Integral Transforms are also useful in various fields and contribute to national development such as radar system, X-ray machine, optical communication, quantum mechanics, signal processing etc.

7) Year wise plan of work and target to be achieved:

First year Plan

1. Collect the statistical data on integral transform by referring books, journals regarding subject.
2. Analysis of results and given data regarding this problem.
3. Define enlarge domain for various transforms.
4. Design new integral transforms and try to solve their properties.

Second year Plan

1. Develop integral transform using collected statistical data.
2. Solve some basic properties on time scale for the given transforms.
3. Obtain some basic theorem for this transform.
4. Development of innovative works regarding this transforms

Third year Plan

1. To solve some applications in selected integral transforms on time scale .
2. To solve differential and integral equations by using new methods on time scale .
3. Try to achieve some applications in real life, engineering problems and Biological sciences.
4. Finally Dissertation will be submitted.

08) References :-

[01] **Alieva T. and Calvo M L. :** "Fractionalization of the Linear Cyclic Transforms", J. Opt Soc. Am., Vol 17, No.12, P 2330-2338, Dec.2000.

[02] **Almeida L.B.:** An introduction to the angular fourier transform, IEEE, (1993), 257-260.

[03] **Almeida L.B.:** The fractional fourier transform and time frequency representation IEEE trans. on signal processing Vol. 42, Nov. (1994), No.11.

[04] **Bhosale B.N. and Choudhary M.S.:** Fractional fourier transform of distributions of compact support, Bull. Cal. Math. Soc., Vol. 94, (2002), No.5, 349-358.

[05] **Chavhan S.B., Borkar V.C.:** Canonical sine transform and their unitary representation.

[06] **Lohman A, Mendolic H.M. and Zalvesky R:** Some important fractional transform for signal processing, optics communication, Vol 125, P 18-20, (1966).

[07] **Moshinky M. Quesne:** Linear canonical transform and their unitary representation, J. Math, Phy., Vol.12, (1971), (No.), 1772-1783.

[08] **Namias V.:** The fractional order fourier transform and its applications in quantum mechanics, I. Inst. Maths. appli., (1980), 25, 241-265.

[09] **Pei, Ding J.:** Eigen function of linear canonical transform, IEEE, trans. sign. proc. Vol.50, Jan.(2002), No.1.

[10] **Pei, Ding J.:** Fractional cosine, sine and hartley transform, trans. on signal processing, Vol.50, No.7, Pub. (2002), P.1661-1680.

[11] **Zayed A.:** A convolution and product theorem for the fractional fourier transform, IEEE sign. proc. letters Vol.5, No.4, (1998), 101-103.

[12] **Zemanian A.H.:** Distribution theory and transform analysis, McGraw Hill, New York, (1965).

[13] **Zemanian A.H.:**Generalized Integral Transform, Inter Science Publisher's New York.

[14] **Watungala GK:** Sumudu transform : a new integral transform to solve differential equation and control engineering problems. Int J Math Edu Sci.Technol. 24(1),35-43(1993) doi;10.1080/0020739930240105.

[15] **Belgacem, FBM, KARABALLI, AA:** Sumudu transform fundamental properties investigation and applications. Journal of applied mathematics and stochastics Analysis 2006,1-23 Article ID 91083 doi1155/JAMSA/2006/91083.

[16] **Eltayeb, H, Kilicman, A fisher:** A new integral transform and associated distributions .Int.Trans. spec Func 21(5),367-379(2010), doi :10.1080/10652460903335061.

[17] **Kilicman, A, Eltayeb, H:** On the applications of laplace and sumudu transforms. J Franklin Institute. 347(5),848-862(2010). Doi:10.1016/j.jfranklin.2010.03.008

[18] **Ahrendt, c:** The laplace transform on time scale .pan Am Math J.19(4),1-36(2009).

[19] **Boher, M, Guesein, GSH:** The convolution on time scale. Abstract Appl Anal 2007, 24(2007), Article id 58373.

[20] **Bohner, M, Peterson, A:** Advance in dynamic Equations on time scale. Birkhauser, Boston, MA(2003).

Mr. Santosh Keshavrao Bokare
(Research Scholar)

Dr.S.B.Chavhan
Head, Dept. of Mathematics Digambarrao Bindu ACS College,Bhokar,
Dist. Nanded, (Maharashtra).